HW 7 Solution

Group Homework Solution

4.49

4.49 Water flows through a duct of square cross section as shown in Fig. P4.49 with a constant, uniform velocity of $V =$ 20 m/s. Consider fluid particles that lie along line $A-B$ at time $t = 0$. Determine the position of these particles, denoted by line $A'-B'$, when $t = 0.20$ s. Use the volume of fluid in the region between lines $A-B$ and $A'-B'$ to determine the flowrate in the duct. Repeat the problem for fluid particles originally along line $C-D$; along line $E-F$. Compare your three answers.

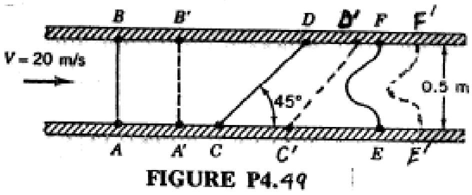


FIGURE P4.49

Since V is constant in time and space, all particles on line AB move a distance $\ell = V\Delta t = (20\frac{m}{s})(0.2s) = 4m$ from $t=0$ to $t=0.2s$

Thus, the volume of $ABA'B'$ is $\forall_{ABA'B'} = (0.5m)^2(4m) = 1.00\ m^3$

so that

$$Q = \frac{\forall_{ABA'B'}}{\Delta t} = \frac{1.00\ m^3}{0.2\,s} = \underline{\underline{5.0\ \frac{m^3}{s}}}$$

Similarly from $t=0$ to $t=0.2s$ the fluid along lines CD and EF move to $C'D'$ and $E'F'$, respectively. Also, $\forall_{CDC'D'} = \forall_{EFE'F'} = \forall_{ABA'B'}$ so that we obtain $Q = \frac{\forall}{\Delta t} = 5.0\frac{m^3}{s}$ regardless which line we consider.

4.51

4.51 In the region just downstream of a sluice gate, the water may develop a reverse flow region as is indicated in Fig. P4.51 and Video V10.5. The velocity profile is assumed to consist of two uniform regions, one with velocity $V_a = 10$ fps and the other with $V_b = 3$ fps. Determine the net flowrate of water across the portion of the control surface at section (2) if the channel is 20 ft wide.

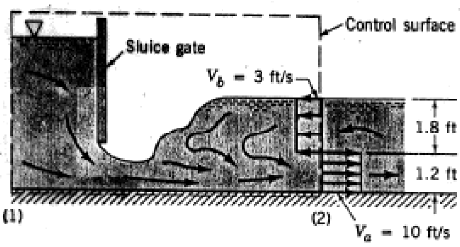
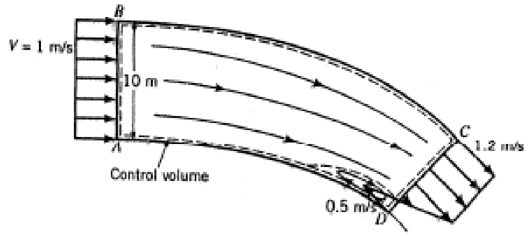


FIGURE P4.51

$$Q = V_a A_a - V_b A_b = (10\tfrac{ft}{s})(1.2ft)(20ft) - (3\tfrac{ft}{s})(1.8ft)(20ft)$$
$$= \underline{\underline{132\ \frac{ft^3}{s}}}$$

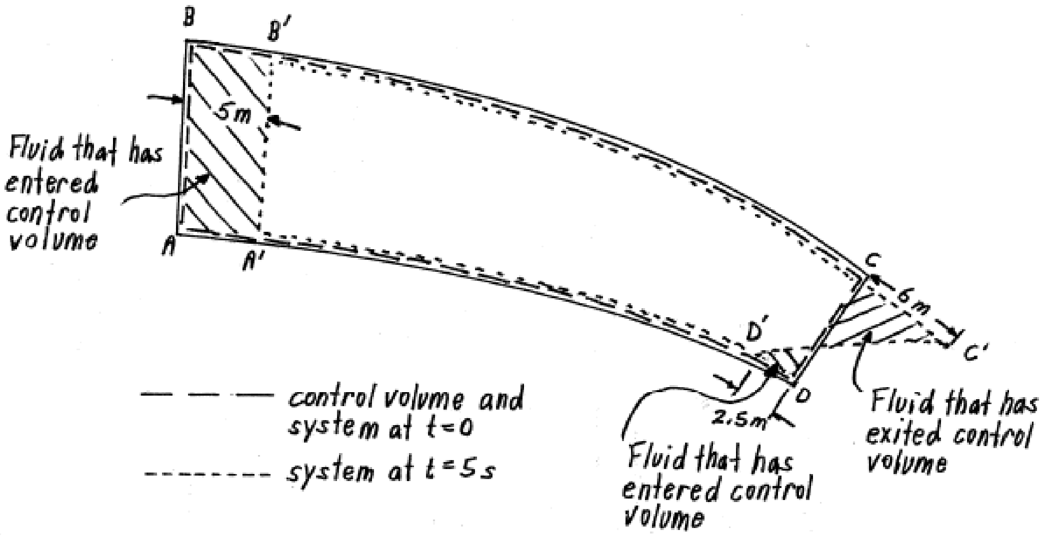
4.54

4.54 Water enters the bend of a river with the uniform velocity profile shown in Fig. P4.54. At the end of the bend there is a region of separation or reverse flow. The fixed control volume *ABCD* coincides with the system at time $t = 0$. Make a sketch to indicate **(a)** the system at time $t = 5$ s and **(b)** the fluid that has entered and exited the control volume in that time period.



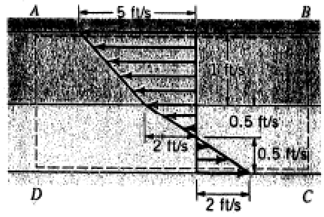
■ FIGURE P4.54

Since the distance the fluid travels in time $\delta t = 5s$ is $\ell = V\delta t$, the fluid at A-B when $t=0$ has traveled $\ell = (1m/s)(5s) = 5m$ when $t = \delta t = 5s$. This is shown in the figure below. Similarly, the fluid across C-D at $t=0$ has moved as indicated when $t = \delta t = 5s$. Thus, the boundary of the system at $t = 5s$ are as show in the figure below. The fluid that entered and exited the control volume in that time period is also shown.



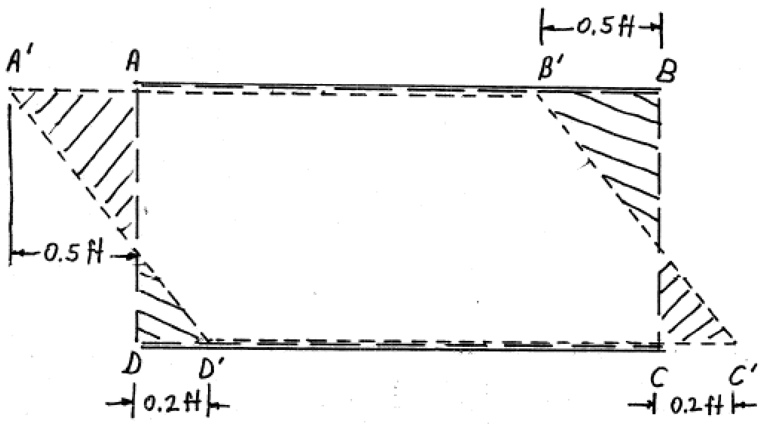
4.57

4.57 Two liquids with different densities and viscosities fill the gap between parallel plates as shown in Fig. P4.57. The top plate moves to the left with a speed of 5 ft/s; the bottom plate moves to the right with a speed of 2 ft/s. The velocity profile consists of two linear segments as indicated. The fixed control volume *ABCD* coincides with the system at time $t = 0$. Make a sketch to indicate (**a**) the system at time $t = 0.1$ s and (**b**) the fluid that has entered and exited the control volume in that time period.



■ FIGURE P4.57

From $t=0$ to $t=0.1s$ the bottom plate (and the liquid that sticks to it) moves $\ell = V\Delta t = (2\ ft/s)(0.1s) = 0.2\ ft$ to the right. Similarly, the top plate moves to the left a distance $\ell = V\Delta t = (5\ ft/s)(0.1s) = 0.5\ ft$. The fluid along lines A-D and B-C also move distances given by $\ell = V\Delta t$. For example, the interface between the 2 liquids moves a distance $\ell = V\Delta t = (2\ ft/s)(0.1s) = 0.2\ ft$. The fluid layer 0.5 ft above the bottom plate has $V=0$ and, therefore, does not move. The corresponding displacement of the fluid originally along A-D and B-C is shown in the figure below.



— — — control volume and system at $t=0$

- - - - - - - - system at $t=0.1s$

////, fluid that exited control volume

\\\\ fluid that entered control volume

5.7

5.7 Water flows along the centerline of a 50-mm-diameter pipe with an average velocity of 10 m/s and out radially between two large circular disks as shown in Fig. P5.7. The disks are parallel and spaced 10 mm apart. Determine the average velocity of the water at a radius of 300 mm in the space between the disks.

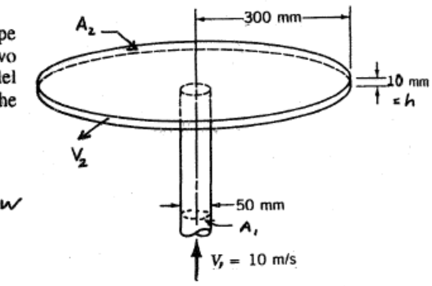


FIGURE P5.7

For steady incompressible flow

$$Q_1 = Q_2$$

or

$$A_1 \bar{V}_1 = A_2 \bar{V}_2$$

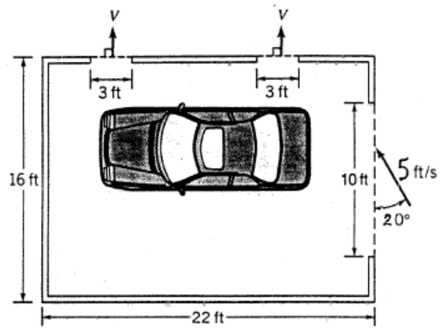
Thus

$$\bar{V}_2 = \frac{A_1}{A_2}\bar{V}_1 = \frac{\pi D_1^2 \;\bar{V}_1}{(4) 2\pi r_2 h} = \frac{(50\text{ mm})^2 (10\text{ m/s})}{(4)(2)(300\text{ mm})(10\text{ mm})}$$

$$\bar{V}_2 = \underline{\underline{1.04\ \frac{m}{s}}}$$

5.5

5.5 The wind blows through a 7 ft × 10 ft garage door opening with a speed of 5 ft/s as shown in Fig. P5.5. Determine the average speed, V, of the air through the two 3 ft × 4 ft openings in the windows.



■ **FIGURE P5.5**

For steady incompressible flow

$$Q_{\substack{garage\\door}} = Q_{window} + Q_{window}$$

or

$$A_{\substack{garage\\door}} V_{\substack{normal\ to\\garage\ door}} = A_{window} V + A_{window} V$$

so the average speed, V, of the air through the two windows is

$$V = \frac{A_{\substack{garage\\door}} V_{normal\ to\ garage\ door}}{2A_{window}} = \frac{(7ft)(10ft)(5\frac{ft}{s})\sin 20^\circ}{2(3ft)(4ft)} = \underline{\underline{4.99\ \frac{ft}{s}}}$$

5.9

5.9 A water jet pump (see Fig. P5.9) involves a jet cross section area of 0.01 m², and a jet velocity of 30 m/s. The jet is surrounded by entrained water. The total cross section area associated with the jet and entrained streams is 0.075 m². These two fluid streams leave the pump thoroughly mixed with an average velocity of 6 m/s through a cross section area of 0.075 m². Determine the pumping rate (i.e., the entrained fluid flowrate) involved in liters/s.

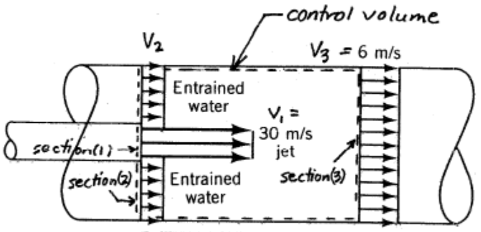


FIGURE P5.9

For steady incompressible flow through the control volume

$$Q_1 + Q_2 = Q_3$$

or

$$\bar{V}_1 A_1 + Q_2 = \bar{V}_3 A_3$$

Thus

$$Q_2 = \bar{V}_3 A_3 - \bar{V}_1 A_1 = \left[\left(6\,\frac{m}{s}\right)(0.075\,m^2) - \left(30\,\frac{m}{s}\right)(0.01\,m^2)\right]\left(1000\,\frac{\text{liters}}{m^3}\right)$$

$$Q_2 = 150\,\frac{\text{liters}}{s}$$

Individual Homework Solution

4.50

4.50 Repeat Problem 4.49 if the velocity profile is linear from 0 to 20 m/s across the duct as shown in Fig. P4.50.

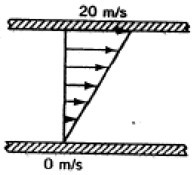


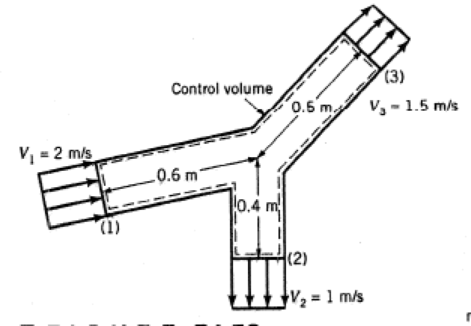
FIGURE P4.50

From $t=0$ to $t=0.1\ s$ the particle initially at B travels a distance $\ell_B = V_B \Delta t = (20\ \frac{m}{s})(0.1\ s) = 2\ m$ as show. Particle A remain fixed since $V_A = 0$. Since the velocity profile is linear, line AB remains straight, but "tilts" as indicated. Thus, the volume of fluid crossing the initial line AB is $\forall_{ABB'} = \frac{1}{2}\ \ell_B\ A = \frac{1}{2}(2\ m)(0.5\ m)^2 = 0.25\ m^3$ so that

$Q = \frac{\forall_{ABB'}}{\Delta t} = \frac{0.25\ m^3}{0.1\ s} = \underline{\underline{2.5\ \frac{m^3}{s}}}$ Since $\forall_{CDD'} = \forall_{EFF'} = V_{ABB'}$ it follows that the same value of Q is obtained regardless which volume is used.

4.56

4.56 Water flows in the branching pipe shown in Fig. P4.56 with uniform velocity at each inlet and outlet. The fixed control volume indicated coincides with the system at time $t = 20$ s. Make a sketch to indicate (a) the boundary of the system at time $t = 20.2$ s, (b) the fluid that left the control volume during that 0.2-s interval, and (c) the fluid that entered the control volume during that time interval.



■ FIGURE P4.56

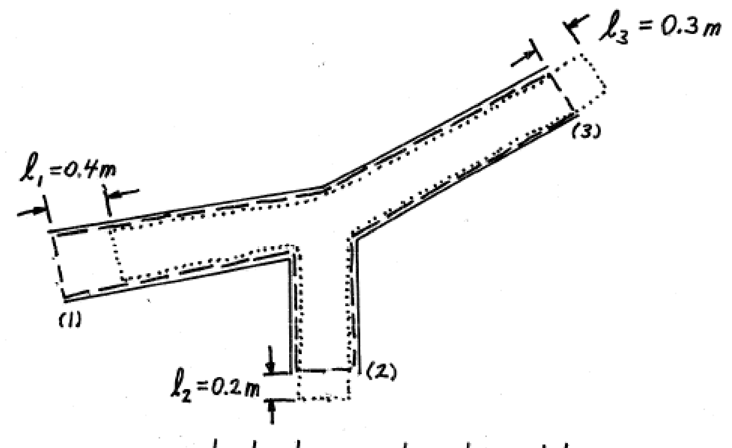
m/s

Since V is constant, the fluid travels a distance $\ell = V\delta t$ in time Δt. Thus, $\ell_1 = V_1\,\delta t = (2\frac{m}{s})(20. - 20)s = 0.4\,m$

$\ell_2 = V_2\,\delta t = (1\frac{m}{s})(20. - 20)s = 0.2\,m$

and $\ell_3 = V_3\,\delta t = (1.5\frac{m}{s})(20. - 20)s = 0.3\,m$

The system at $t = 20.2s$ and the fluid that has entered or exited the control volume are indicated in the figure below.

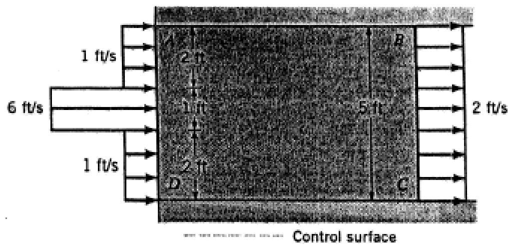


— — — control volume and system at $t = 20s$

.... system at $t = 20.2s$

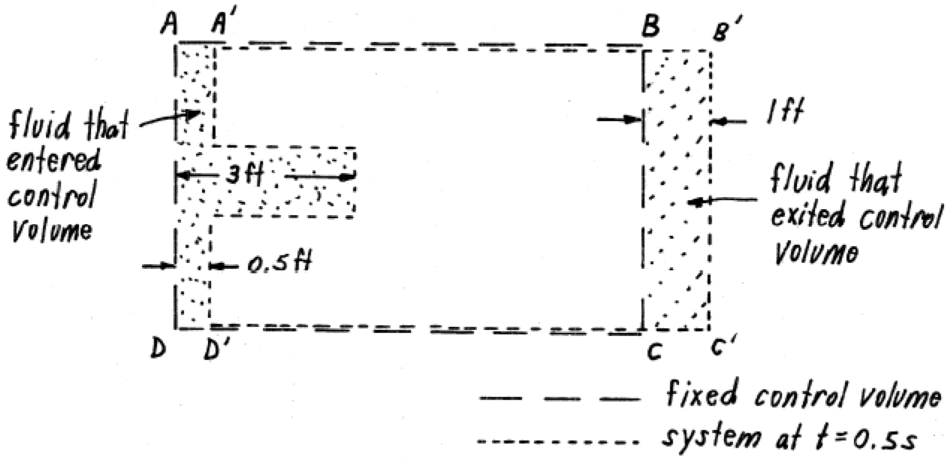
4.59

4.59 Water enters a 5-ft-wide, 1-ft-deep channel as shown in Fig. P4.59. Across the inlet the water velocity is 6 ft/s in the center portion of the channel and 1 ft/s in the remainder of it. Farther downstream the water flows at a uniform 2 ft/s velocity across the entire channel. The fixed control volume *ABCD* coincides with the system at time $t = 0$. Make a sketch to indicate (a) the system at time $t = 0.5$ s and (b) the fluid that has entered and exited the control volume in that time period.



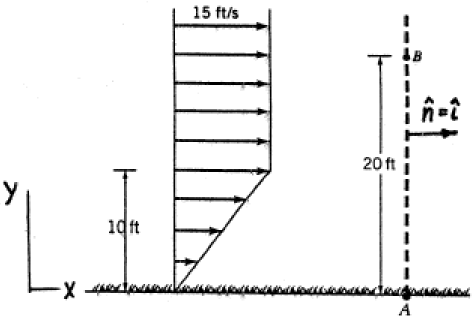
■ FIGURE P4.59

During the $t = 0.5$ s time interval the fluid that was along line BC at time $t=0$ has moved to the right a distance $\ell = V\,t = 2\frac{ft}{s}(0.5s) = 1\,ft$. Similarly, portions of the fluid along line AD have moved $\ell = 1\frac{ft}{s}(0.5s) = 0.5\,ft$ and $\ell = 6\frac{ft}{s}(0.5s) = 3\,ft$. This assumes the $1\frac{ft}{s}$ and $6\frac{ft}{s}$ fluid streams do not mix or intermingle during the 0.5 s time interval. See figure below.



4.61

4.61 The wind blows across a field with an approximate velocity profile as shown in Fig. P4.61. Use Eq. 4.16 with the parameter b equal to the velocity to determine the momentum flowrate across the vertical surface A–B, which is of unit depth into the paper.



■ FIGURE P4.61

$$\vec{\dot{B}}_{AB} = \int_{AB} \rho b \vec{V}\cdot\hat{n}\, dA = \int_{AB} \rho \vec{V}\ \vec{V}\cdot\hat{n}\, dA = \rho \int_{y=0}^{y=20\,ft} (V\hat{i})[(V\hat{i})\cdot\hat{i}](1\,ft)\,dy$$

$$= \rho\hat{i}\int_0^{20} V^2\, dy$$

But, $V = \frac{15}{10}y\ \frac{ft}{s}$ for $0 \le y \le 10\,ft$ (i.e., $V=0$ at $y=0$; $V = 15\frac{ft}{s}$ at $y=10$)
and $V = 15\frac{ft}{s}$ for $y \ge 10\,ft$

Thus,

$$\vec{\dot{B}}_{AB} = \rho\hat{i}\left[\int_0^{10}\left(\frac{15}{10}y\right)^2 dy + \int_{10}^{20}(15)^2 dy\right] = \rho\hat{i}\left[2.25\,\frac{y^3}{3}\Big|_0^{10} + 225\,y\Big|_0^{10}\right]$$

$$= 0.00238\,\frac{slugs}{ft^3}\left[750\,\frac{ft^4}{s^2} + 2250\,\frac{ft^4}{s^2}\right]\hat{i}$$

$$= 7.14\hat{i}\,\frac{slug\ ft}{s^2}$$

5.1

5.1 Water enters a conical diffusing passage (see Fig. P5.1) with an average velocity of 10 ft/s. If the entrance cross section area is 1 ft², how large should the diffuser exit area be to reduce the average velocity level to 1 ft/s?

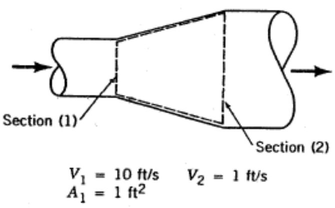


FIGURE P5.1

For steady incompressible flow between sections (1) and (2)

$$Q_1 = Q_2$$

or

$$A_1\bar{V}_1 = A_2\bar{V}_2$$

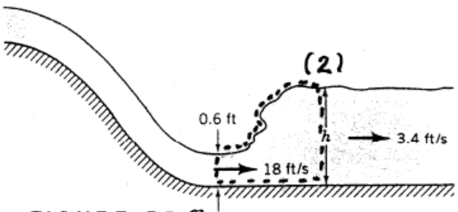
So

$$A_2 = \frac{A_1\bar{V}_1}{\bar{V}_2} = \frac{(1\ ft^2)(10\ \frac{ft}{s})}{(1\ \frac{ft}{s})}$$

$$\underline{\underline{A_2 = 10\ ft^2}}$$

5.8

5.8 A hydraulic jump (see Video V10.5) is in place downstream from a spill-way as indicated in Fig. P5.8. Upstream of the jump, the depth of the stream is 0.6 ft and the average stream velocity is 18 ft/s. Just downstream of the jump, the average stream velocity is 3.4 ft/s. Calculate the depth of the stream, h, just downstream of the jump.



■ FIGURE P5.8

For steady incompressible flow between sections (1) and (2)

$$Q_1 = Q_2$$

or

$$\bar{V}_1 A_1 = \bar{V}_2 A_2$$

Thus

$$\bar{V}_1 h_1 = \bar{V}_2 h_2$$

and

$$h_2 = \frac{\bar{V}_1 h_1}{\bar{V}_2} = \frac{(18\ \frac{ft}{s})(0.6\ ft)}{(3.4\ \frac{ft}{s})} = \underline{\underline{3.18\ ft}}$$

5.13

5.13 Two rivers merge to form a larger river as shown in Fig. P5.13. At a location downstream from the junction (before the two streams completely merge), the nonuniform velocity profile is as shown. Determine the value of V.

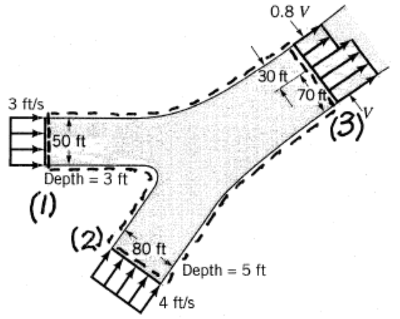


FIGURE P5.13

Use the control volume shown within broken lines in the sketch above. We note that $\dot{m} = \rho A V$ and from the conservation of mass principle we get

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3 = \dot{m}_{0.8V} + \dot{m}_V$$

Thus

$$\rho A_1 V_1 + \rho A_2 V_2 = \rho A_{0.8V}\, 0.8 V + \rho A_V V$$

and

$$V = \frac{A_1 V_1 + A_2 V_2}{A_{0.8V}(0.8) + A_V} = \frac{(50 ft)(3 ft)(3 \frac{ft}{s}) + (80 ft)(5 ft)(4 \frac{ft}{s})}{(30 ft)(6 ft)(0.8) + (70 ft)(6 ft)}$$

$$V = \underline{\underline{3.63}} \frac{ft}{s}$$